

AN APPLICATION OF RADON AND WAVELET TRANSFORMS FOR IMAGE FEATURE EXTRACTION

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ABSTRACT

In this paper we proposed wavelet and Radon for the rotation and translation invariant image transforms analysis and their use for image enhancement and features extraction. Main aim of this paper is to use two-dimensional Radon and wavelet transforms to form fundamental mathematical tools in these areas. Results are verified in the MATLAB environment both for data and for analysis of biomedical images.

KEYWORDS: Radon and Wavelet, Transformation to Detect Lines and Curves in Pictures

INTRODUCTION

This paper allows features extraction by blocks of Radon transform, wavelet transform and blocks of image preprocessing. Individual features are obtained by connection of these blocks using a wavelet decomposition block into the second level. Two features obtained by this decomposition are sum of squared image component coefficients evaluated in the first and the second decomposition level by high-pass filters both for image columns and rows. Results of features' variance with application of different methods are displayed both graphically and in the tables.

The Radon transform is named after the Austrian mathematician Johann Karl August Radon (December 16, 1887 – May 25, 1956). The main application of the Radon transform is CAT scans, where the inverse Radon transform is applied. The Radon transform can also be used for line detection. Radon transform forming a very important mathematical tool used in tomography is based upon works of Johann Radon born in 1887 Litomerice. His doctoral dissertation has been defended in Vienna in 1910 and his most appreciated works were devoted to integral geometry. The Radon transform [1] belonging to this category introduced in 1917 is defined as a collection of 1D projections around an object at angle intervals θ . The Radon transform of a two-dimensional (2-D) function $f(x, y)$ is defined as $R(\theta, r)R(\theta, r)[f(x, y)] =$

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x, y) \delta(r - x \cos \theta - y \sin \theta) dx dy \quad (1)$$

where r is the perpendicular distance of a line from the origin and θ is the angle formed by the distance vector.

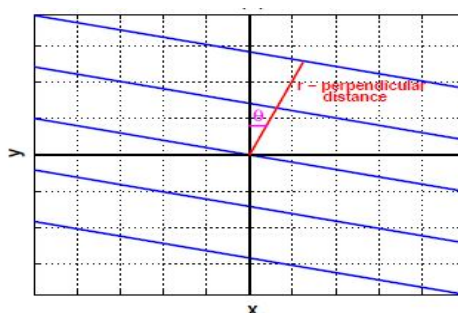


Figure 1(a)

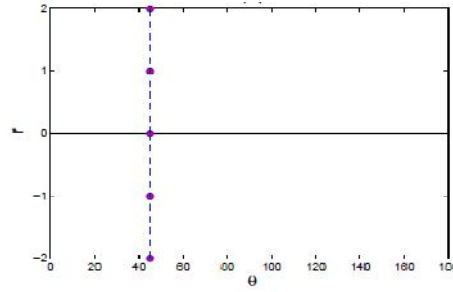


Figure 1(b)

Figure 1: Figure presents (a) an example of a set of parallel lines for a chosen $\theta = 45^\circ$ in the (x, y) plane and (b) the localization of corresponding points in the (θ, r) plane in which the discrete Radon transform is evaluated.

For the constant value of Θ the set of parallel lines for different values of r are presented in Fig. 1(a). The parallel lines are used for the integration of the given image. The plane (x, y) is transformed in this way to the plane (θ, r) . The transformation proceeds by integration of the given image along parallel lines in the plane (x, y) and resulting value is then marked in the graph as a point for a given θ and r as depicted in Fig. 1(b). Each point has a different intensity of color, depending on its value, having value 0 corresponding to black and 1 corresponding to white color presented in Fig. 3(b). A discrete Radon transform called Hough transform has been introduced in 1972 by R. Duda and P. Hart [3] as a tool for image features extraction.

Simulation of Radon Transform

In the Simulink environment there is no block for the Radon transform. A general block called "Matlab function" can be used instead. This block has a single input and single output. Parameters of this block include:

- Name of existing function of Matlab library or name of the created function as M-file
- Output dimensions specified for returned single value
- Choice to Collapse 2-D results to 1-D

The Matlab function or M-file use every "Matlab function" block for processing of the input value. Fig. 2 presents block diagram of the direct and inverse Radon transform of MR image and visualization of Radon transform image. Input image is loaded as a constant and output variable is frame-based. Input image and images after transformation are visualized in the matrix viewer presented in Fig. 3 and sent to workspace.

Direct and Inverse Radon Transform

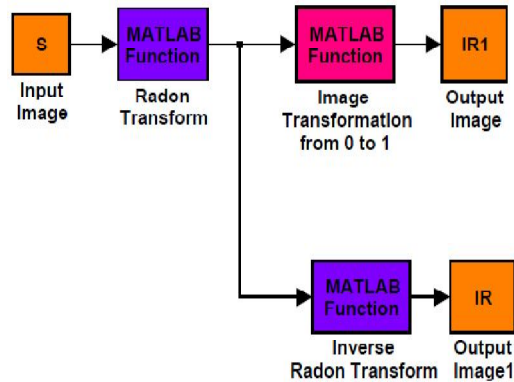


Figure 2: Block Diagram of the Proposed Technique

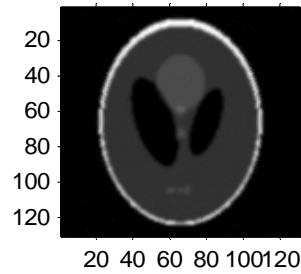


Figure 3 (a)

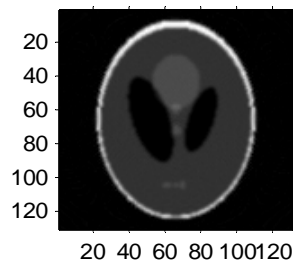


Figure 3 (b)

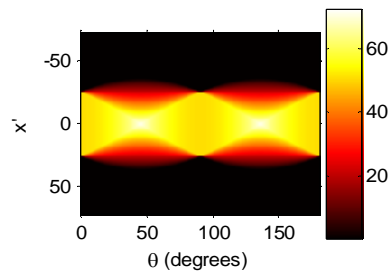


Figure 3(c)

Figure 3: Visualization of (a) input MR Image Rotated by $\theta = 90^\circ$, its (b) Radon Transform Depicted as Points for $\theta = 0^\circ - 180^\circ$ and for the Same r for each θ , and (c) Inverse Radon Transform

Radon Transform to Detect Lines

The Radon transform is closely related to a common computer vision operation known as the Hough transform. You can use the radon function to implement a form of the Hough transform used to detect straight lines

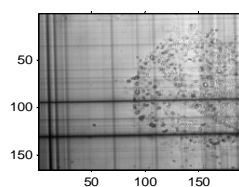


Figure 4 (a): Original Image

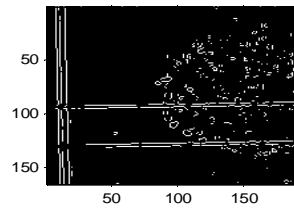


Figure 4(b): Edge Image

Principles of Image Wavelet Decomposition

Wavelet functions used for signal analysis are derived from the initial function $W(t)$ forming basis for the set of functions

$$W_{a,b}(t) = \frac{1}{\sqrt{a}} W\left(\frac{1}{a}(t-b)\right)$$

for discrete parameters of dilation $a=2^m$ and translation $b=k \cdot 2^m$. Wavelet dilation, which is closely related to spectrum compression, enables local and global signal analysis. The principle of signal and image decomposition for resolution enhancement is presented in Fig. 4. The wavelet transform has gained a great deal of interest due to its time localization and multiresolution properties. Fourier transforms (FT) lack time localization as frequency components are attributed to the entire time signal and not to specific parts of it. Windowed Fourier Transforms (WFT) achieves this localization by using a window function that limits the duration of the analyzed signal segment. WFT uses fixed size windows that cannot be adjusted to suite the speed of the changing phenomena observed in the input signals.

Wavelets solve this problem by using the so called mother wavelet which can be scaled and translated to achieve both time localization and multi-resolution.

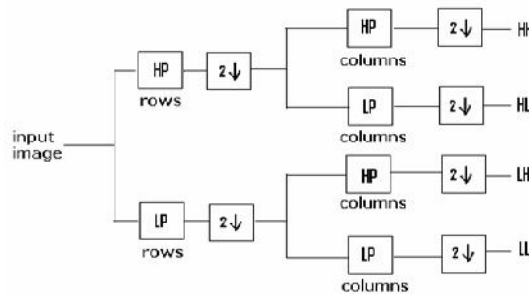


Figure 5: Wavelet Decomposition

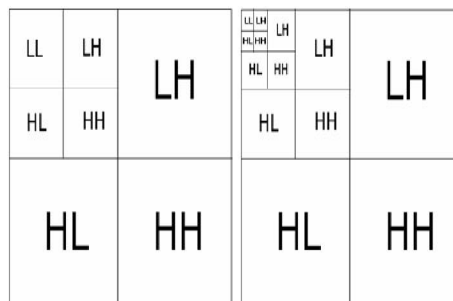


Figure 6: 2-Level and 4-Level Decomposition

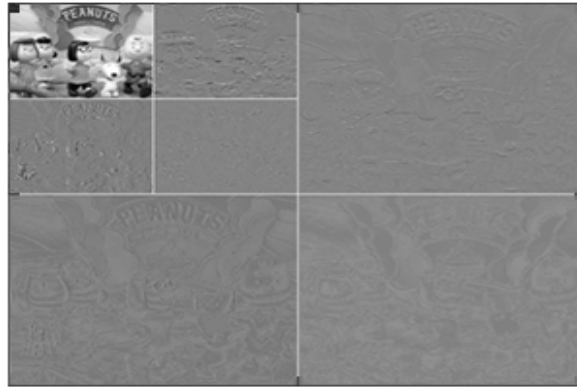


Figure 7: 2-Level Decomposition of Reference Image

The decomposition stage results in this way in four images representing all combinations of low-pass and high-pass initial image matrix. The reconstruction stage includes row upsampling at first and row convolution in stage $R.1$. The corresponding images are then summed. The final step $R.2$ assumes column upsampling and convolution with reconstruction filters followed by summation of the results again. In the case of one-dimensional signal processing, steps $D.2$ and $R.1$ are omitted.

Simulation of Wavelet Transform in Simulink Environment

Wavelet transform diagram was created with blocks of Simulink library. Block "DWT" computes the discrete wavelet transform using a filter bank with specified highpass and lowpass filters. The filters can be user-defined or formed by wavelets of the Wavelet Toolbox. The output is set to 'Multiple ports'. It enables to see each sub band as a frame-based vector or matrix. The common block "Transpose" enables matrix transposition. In our diagram it enables matrix transposition after column downsampling to proceed row decomposition. We transpose matrix after the row decomposition to visualize matrix right. Diagram for one decomposition and reconstruction levels is presented in Fig. 5.

The whole diagram for image decomposition into the second level and its reconstruction is presented in Fig. 6.

Block diagrams mentioned above have been created to obtain definition of features of rotated images. We compare the standard deviation (STD) of the sum of squared diagonal DWT transform coefficients in the first and the second decomposition levels using MR images obtained by rotation from 0 to 180 degrees with step 10° using (i) diagram with the plain DWT, (ii) diagram for the Radon transform (RT) followed by the DWT diagram. Computed values of STD are displayed in Tab. 1.

Table 1: Table Presents STD Computed from Rotated MR Image Features

STD of MR Image Features		
	Feature-1	Feature-2
DWT	0.0013	0.0254
RT-DWT	2.97×10^{-5}	0.0023

RESULTS

Thanks to the objective confrontation of STDs, Tab 1 is the bright example that the Radon transform is a powerful tool expressively contributing to image analysis. The improvement of the STD between the plain DWT and RT-DWT by an order has been verified. We achieved also a small improvement by denoising of the magnetic resonance image.

Therefore image enhancement is very desirable here. We also tested with simulink of MATLAB(fig.8 &fig.9) and also using other images like a woman image(Fig.10 & fig.11) which is in built in MATLAB. Image preprocessing allows further research devoted to the optimization of wavelet coefficients thresholding to denoise the original image. The proposed method of image features extraction allows the estimation of the rotation invariant image features and moreover it is very flexible as it allows the use of different wavelet functions and different rotation steps in case of the Radon transform. It is assumed that further studies will be devoted to feature based image segmentation and further methods of rotation and translation invariant feature selection using appropriate image transforms.

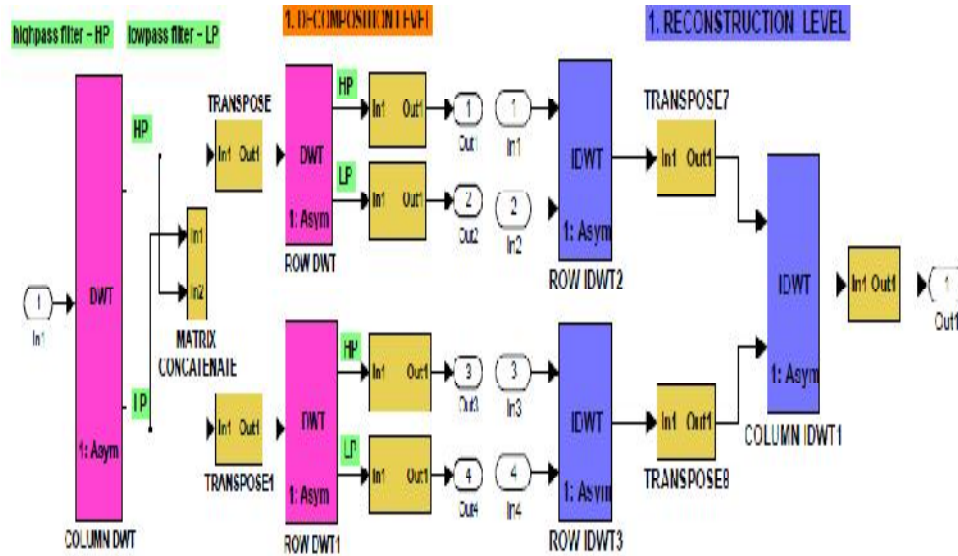


Figure 8: Figure Presents Connection of Individual Simulink Blocks Which Create one Level of Wavelet Decomposition and Reconstruction.

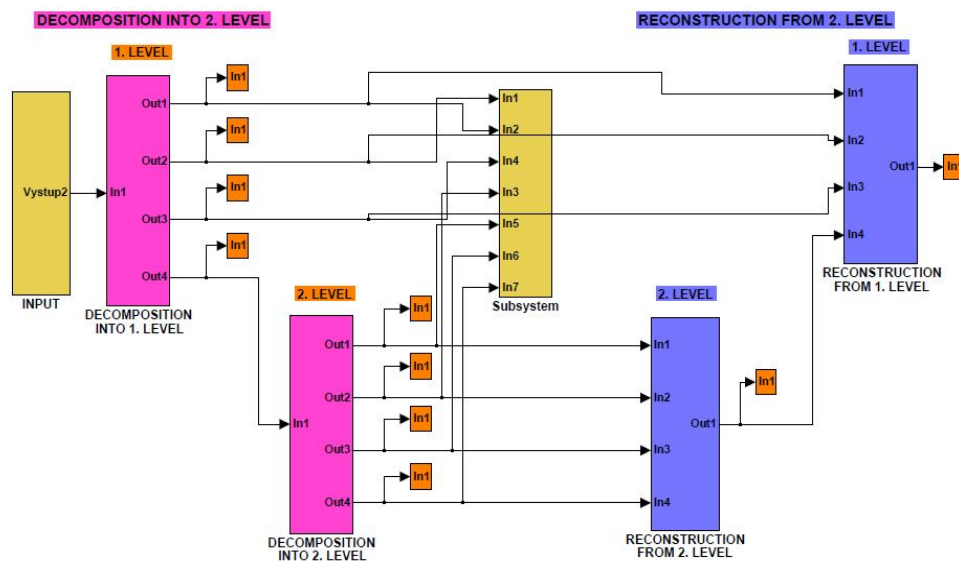


Figure 9: The Simulink Diagram for Wavelet Transform Decomposition and Reconstruction Applied to two Levels Diagram was Applied to a MR Image. Resulting Images from the Matrix Viewer are Depicted in Fig. 7.

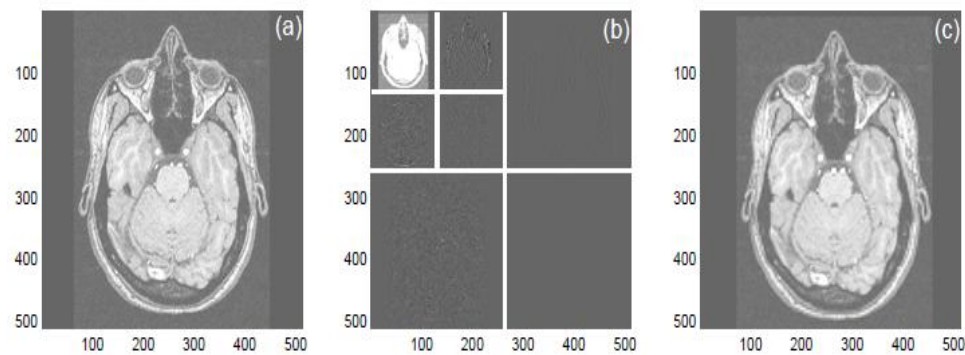


Figure 10: Visualization of (a) Input MR Image, Its (b) Wavelet Decomposition, and (c) Image Wavelet Reconstruction

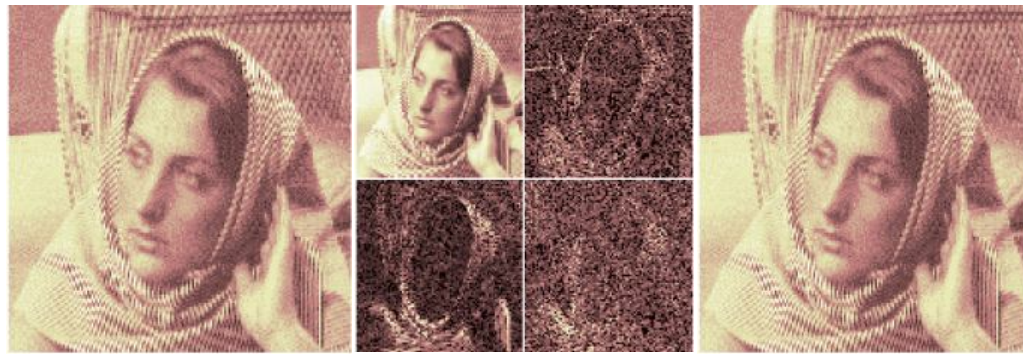


Figure 11: Visualization of (a) Input Woman Image, Its (b) Wavelet Decomposition, and (c) Image Wavelet Reconstruction

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